

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

- Q.1 Let  $\mathbb{R}$  denote the set of all real numbers. Let  $a_i, b_i \in \mathbb{R}$  for  $i \in \{1, 2, 3\}$ . Define the functions  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$ , and  $h: \mathbb{R} \rightarrow \mathbb{R}$  by

$$\begin{aligned} f(x) &= a_1 + 10x + a_2x^2 + a_3x^3 + x^4, \\ g(x) &= b_1 + 3x + b_2x^2 + b_3x^3 + x^4, \\ h(x) &= f(x+1) - g(x+2). \end{aligned}$$

If  $f(x) \neq g(x)$  for every  $x \in \mathbb{R}$ , then the coefficient of  $x^3$  in  $h(x)$  is

|     |    |
|-----|----|
| (A) | 8  |
| (B) | 2  |
| (C) | -4 |
| (D) | -6 |

- Q.2 Three students  $S_1, S_2$ , and  $S_3$  are given a problem to solve. Consider the following events:

$U$ : At least one of  $S_1, S_2$ , and  $S_3$  can solve the problem,  
 $V$ :  $S_1$  can solve the problem, given that neither  $S_2$  nor  $S_3$  can solve the problem,  
 $W$ :  $S_2$  can solve the problem and  $S_3$  cannot solve the problem,  
 $T$ :  $S_3$  can solve the problem.

For any event  $E$ , let  $P(E)$  denote the probability of  $E$ . If

$$P(U) = \frac{1}{2}, \quad P(V) = \frac{1}{10}, \quad \text{and} \quad P(W) = \frac{1}{12},$$

then  $P(T)$  is equal to

|     |                 |     |               |     |                 |     |               |
|-----|-----------------|-----|---------------|-----|-----------------|-----|---------------|
| (A) | $\frac{13}{36}$ | (B) | $\frac{1}{3}$ | (C) | $\frac{19}{60}$ | (D) | $\frac{1}{4}$ |
|-----|-----------------|-----|---------------|-----|-----------------|-----|---------------|

Q.3

Let  $\mathbb{R}$  denote the set of all real numbers. Define the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

|     |                                                                                                                              |
|-----|------------------------------------------------------------------------------------------------------------------------------|
| (A) | The function $f$ is <b>NOT</b> differentiable at $x = 0$                                                                     |
| (B) | There is a positive real number $\delta$ , such that $f$ is a decreasing function on the interval $(0, \delta)$              |
| (C) | For any positive real number $\delta$ , the function $f$ is <b>NOT</b> an increasing function on the interval $(-\delta, 0)$ |
| (D) | $x = 0$ is a point of local minima of $f$                                                                                    |

Q.4

Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix  $X$  be denoted by  $X^T$ . Then the number of  $3 \times 3$  invertible matrices  $Q$  with integer entries, such that

$$Q^{-1} = Q^T \text{ and } PQ = QP,$$

is

|     |    |     |   |     |    |     |    |
|-----|----|-----|---|-----|----|-----|----|
| (A) | 32 | (B) | 8 | (C) | 16 | (D) | 24 |
|-----|----|-----|---|-----|----|-----|----|

**SECTION 2 (Maximum Marks: 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;  
*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;  
*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;  
*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then  
 choosing ONLY (A), (B) and (D) will get +4 marks;  
 choosing ONLY (A) and (B) will get +2 marks;  
 choosing ONLY (A) and (D) will get +2 marks;  
 choosing ONLY (B) and (D) will get +2 marks;  
 choosing ONLY (A) will get +1 mark;  
 choosing ONLY (B) will get +1 mark;  
 choosing ONLY (D) will get +1 mark;  
 choosing no option (i.e. the question is unanswered) will get 0 marks; and  
 choosing any other combination of options will get -2 marks.

Q.5 Let  $L_1$  be the line of intersection of the planes given by the equations

$$2x + 3y + z = 4 \quad \text{and} \quad x + 2y + z = 5.$$

Let  $L_2$  be the line passing through the point  $P(2, -1, 3)$  and parallel to  $L_1$ . Let  $M$  denote the plane given by the equation

$$2x + y - 2z = 6.$$

Suppose that the line  $L_2$  meets the plane  $M$  at the point  $Q$ . Let  $R$  be the foot of the perpendicular drawn from  $P$  to the plane  $M$ .

Then which of the following statements is (are) TRUE?

|     |                                                                                                        |
|-----|--------------------------------------------------------------------------------------------------------|
| (A) | The length of the line segment $PQ$ is $9\sqrt{3}$                                                     |
| (B) | The length of the line segment $QR$ is 15                                                              |
| (C) | The area of $\Delta PQR$ is $\frac{3}{2}\sqrt{234}$                                                    |
| (D) | The acute angle between the line segments $PQ$ and $PR$ is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ |

Q.6

Let  $\mathbb{N}$  denote the set of all natural numbers, and  $\mathbb{Z}$  denote the set of all integers. Consider the functions  $f: \mathbb{N} \rightarrow \mathbb{Z}$  and  $g: \mathbb{Z} \rightarrow \mathbb{N}$  defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd,} \\ (4-n)/2 & \text{if } n \text{ is even,} \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define  $(g \circ f)(n) = g(f(n))$  for all  $n \in \mathbb{N}$ , and  $(f \circ g)(n) = f(g(n))$  for all  $n \in \mathbb{Z}$ .

Then which of the following statements is (are) TRUE?

|     |                                                                      |
|-----|----------------------------------------------------------------------|
| (A) | $g \circ f$ is <b>NOT</b> one-one and $g \circ f$ is <b>NOT</b> onto |
| (B) | $f \circ g$ is <b>NOT</b> one-one but $f \circ g$ is onto            |
| (C) | $g$ is one-one and $g$ is onto                                       |
| (D) | $f$ is <b>NOT</b> one-one but $f$ is onto                            |

Q.7

Let  $\mathbb{R}$  denote the set of all real numbers. Let  $z_1 = 1 + 2i$  and  $z_2 = 3i$  be two complex numbers, where  $i = \sqrt{-1}$ . Let

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}.$$

Then which of the following statements is (are) TRUE?

|     |                                                                       |
|-----|-----------------------------------------------------------------------|
| (A) | $S$ is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$ |
| (B) | $S$ is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$   |
| (C) | $S$ is a circle with radius $\frac{\sqrt{2}}{3}$                      |
| (D) | $S$ is a circle with radius $\frac{2\sqrt{2}}{3}$                     |

**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks* : +4 If **ONLY** the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

- Q.8 Let the set of all relations  $R$  on the set  $\{a, b, c, d, e, f\}$ , such that  $R$  is reflexive and symmetric, and  $R$  contains exactly 10 elements, be denoted by  $\mathcal{S}$ .

Then the number of elements in  $\mathcal{S}$  is \_\_\_\_\_.

- Q.9 For any two points  $M$  and  $N$  in the  $XY$ -plane, let  $\overrightarrow{MN}$  denote the vector from  $M$  to  $N$ , and  $\vec{0}$  denote the zero vector. Let  $P, Q$  and  $R$  be three distinct points in the  $XY$ -plane. Let  $S$  be a point inside the triangle  $\Delta PQR$  such that

$$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}.$$

Let  $E$  and  $F$  be the mid-points of the sides  $PR$  and  $QR$ , respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is \_\_\_\_\_.

- Q.10 Let  $S$  be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in  $S$ , but 0210222 is **NOT** in  $S$ . Then the number of elements  $x$  in  $S$  such that at least one of the digits 0 and 1 appears exactly twice in  $x$ , is equal to \_\_\_\_\_.

Q.11 Let  $\alpha$  and  $\beta$  be the real numbers such that

$$\lim_{x \rightarrow 0} \frac{1}{x^3} \left( \frac{\alpha}{2} \int_0^x \frac{1}{1-t^2} dt + \beta x \cos x \right) = 2.$$

Then the value of  $\alpha + \beta$  is \_\_\_\_\_.

Q.12 Let  $\mathbb{R}$  denote the set of all real numbers. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) > 0$  for all  $x \in \mathbb{R}$ , and  $f(x+y) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$ .

Let the real numbers  $a_1, a_2, \dots, a_{50}$  be in an arithmetic progression. If  $f(a_{31}) = 64f(a_{25})$ , and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is \_\_\_\_\_.

Q.13 For all  $x > 0$ , let  $y_1(x)$ ,  $y_2(x)$ , and  $y_3(x)$  be the functions satisfying

$$\begin{aligned} \frac{dy_1}{dx} - (\sin x)^2 y_1 &= 0, & y_1(1) &= 5, \\ \frac{dy_2}{dx} - (\cos x)^2 y_2 &= 0, & y_2(1) &= \frac{1}{3}, \\ \frac{dy_3}{dx} - \left(\frac{2-x^3}{x^3}\right) y_3 &= 0, & y_3(1) &= \frac{3}{5e}, \end{aligned}$$

respectively. Then

$$\lim_{x \rightarrow 0^+} \frac{y_1(x)y_2(x)y_3(x) + 2x}{e^{3x} \sin x}$$

is equal to \_\_\_\_\_.

**SECTION 4 (Maximum Marks: 12)**

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks* : +4 **ONLY** if the option corresponding to the correct combination is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

Q.14 Consider the following frequency distribution:

|           |   |       |       |   |   |    |    |
|-----------|---|-------|-------|---|---|----|----|
| Value     | 4 | 5     | 8     | 9 | 6 | 12 | 11 |
| Frequency | 5 | $f_1$ | $f_2$ | 2 | 1 | 1  | 3  |

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let  $\alpha$  denote the mean deviation about the mean,  $\beta$  denote the mean deviation about the median, and  $\sigma^2$  denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

| List-I                        | List-II |
|-------------------------------|---------|
| (P) $7f_1 + 9f_2$ is equal to | (1) 146 |
| (Q) $19\alpha$ is equal to    | (2) 47  |
|                               | (3) 48  |
| (R) $19\beta$ is equal to     | (4) 145 |
| (S) $19\sigma^2$ is equal to  | (5) 55  |

|     |                                                                                         |
|-----|-----------------------------------------------------------------------------------------|
| (A) | (P) $\rightarrow$ (5) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (2) (S) $\rightarrow$ (4) |
| (B) | (P) $\rightarrow$ (5) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (3) (S) $\rightarrow$ (1) |
| (C) | (P) $\rightarrow$ (5) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (2) (S) $\rightarrow$ (1) |
| (D) | (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (5) (S) $\rightarrow$ (4) |

Q.15

Let  $\mathbb{R}$  denote the set of all real numbers. For a real number  $x$ , let  $[x]$  denote the greatest integer less than or equal to  $x$ . Let  $n$  denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I****List-II**

(P) The minimum value of  $n$  for which the function

$$f(x) = \left\lfloor \frac{10x^3 - 45x^2 + 60x + 35}{n} \right\rfloor$$

is continuous on the interval  $[1, 2]$ , is

(1) 8

(2) 9

(Q) The minimum value of  $n$  for which

$g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$ ,  
 $x \in \mathbb{R}$ , is an increasing function on  $\mathbb{R}$ , is

(R) The smallest natural number  $n$  which is greater than 5, such that  $x = 3$  is a point of local minima of

$h(x) = (x^2 - 9)^n(x^2 + 2x + 3)$ ,  
 is

(3) 5

(S) Number of  $x_0 \in \mathbb{R}$  such that

$$l(x) = \sum_{k=0}^4 \left( \sin|x - k| + \cos \left| x - k + \frac{1}{2} \right| \right),$$

$x \in \mathbb{R}$ , is **NOT** differentiable at  $x_0$ , is

(4) 6

(5) 10

|     |                       |                       |                       |                       |
|-----|-----------------------|-----------------------|-----------------------|-----------------------|
| (A) | (P) $\rightarrow$ (1) | (Q) $\rightarrow$ (3) | (R) $\rightarrow$ (2) | (S) $\rightarrow$ (5) |
| (B) | (P) $\rightarrow$ (2) | (Q) $\rightarrow$ (1) | (R) $\rightarrow$ (4) | (S) $\rightarrow$ (3) |
| (C) | (P) $\rightarrow$ (5) | (Q) $\rightarrow$ (1) | (R) $\rightarrow$ (4) | (S) $\rightarrow$ (3) |
| (D) | (P) $\rightarrow$ (2) | (Q) $\rightarrow$ (3) | (R) $\rightarrow$ (1) | (S) $\rightarrow$ (5) |



Q.16

Let  $\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$ , and  $\vec{u}$  and  $\vec{v}$  be two vectors, such that  $\vec{u} \times \vec{v} = \vec{w}$  and  $\vec{v} \times \vec{w} = \vec{u}$ . Let  $\alpha, \beta, \gamma$ , and  $t$  be real numbers such that

$$\vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \quad -t\alpha + \beta + \gamma = 0, \quad \alpha - t\beta + \gamma = 0, \quad \text{and} \quad \alpha + \beta - t\gamma = 0.$$

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I****List-II**

(P)  $|\vec{v}|^2$  is equal to

(1) 0

(Q) If  $\alpha = \sqrt{3}$ , then  $\gamma^2$  is equal to

(2) 1

(R) If  $\alpha = \sqrt{3}$ , then  $(\beta + \gamma)^2$  is equal to

(3) 2

(S) If  $\alpha = \sqrt{2}$ , then  $t + 3$  is equal to

(4) 3

(5) 5

|     |                       |                       |                       |                       |
|-----|-----------------------|-----------------------|-----------------------|-----------------------|
| (A) | (P) $\rightarrow$ (2) | (Q) $\rightarrow$ (1) | (R) $\rightarrow$ (4) | (S) $\rightarrow$ (5) |
| (B) | (P) $\rightarrow$ (2) | (Q) $\rightarrow$ (4) | (R) $\rightarrow$ (3) | (S) $\rightarrow$ (5) |
| (C) | (P) $\rightarrow$ (2) | (Q) $\rightarrow$ (1) | (R) $\rightarrow$ (4) | (S) $\rightarrow$ (3) |
| (D) | (P) $\rightarrow$ (5) | (Q) $\rightarrow$ (4) | (R) $\rightarrow$ (1) | (S) $\rightarrow$ (3) |